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INTEGRALS IN THE TOPLOSKY RIB FORMULATION

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ADMINISTRATIVE INFORMATION

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BACKGROUND

The Toplosky solution (Toplosky and Vogelsong, 1993) for scattering from a rib (or ribs), based on the approach by Stepanishen (1978), is expressed in terms of several response Green's functions. The response Green's functions (or plate cross-admittance functions) are

$$\Gamma_0 \equiv G_F(\varkappa_m | \varkappa_n) = \frac{k}{2\pi} \int_{-\infty}^{+\infty} Y(\xi) e^{ik\xi(x_m - x_n)} d\xi \, ,$$

$$\Gamma_1 \equiv G_M(\varkappa_m | \varkappa_n) = \frac{k}{2\pi} \int_{-\infty}^{+\infty} ik\xi \, Y(\xi) e^{ik\xi(x_m - x_n)} d\xi \, ,$$

$$\Gamma_2 \equiv G'_M(\varkappa_m | \varkappa_n) = \frac{k}{2\pi} \int_{-\infty}^{+\infty} (ik\xi)^2 \, Y(\xi) e^{ik\xi(x_m - x_n)} d\xi \, ,$$

where k is the wavenumber and $Y(\xi)$ is the admittance of a fluid loaded plate.

$$Y(\xi) = \frac{1}{Z_p(\xi) + Z_a(\xi)} \, ,$$

where

$$Z_a(\xi) = \frac{\rho c}{\sqrt{1 - \xi^2}}$$

is the acoustic radiation impedance, with ρ and c the fluid density and sound speed.

$$Z_p(\xi) = i\omega \rho_p h \left[\Omega^2 \xi^4 - 1 \right]$$

is the impedance of an unstiffened Bernoulli–Euler (thin) plate, where ρ_p and h are the density and thickness of the plate and $\Omega = \frac{\omega}{\omega_c}$. The coincidence frequency

$$\omega_c = \frac{\sqrt{12} c^2}{h c_p} \, ,$$

with $c_p = \sqrt{\dfrac{E}{\rho_p(1 - v^2)}}$. E and v are the Young's modulus and Poisson's ratio of the plate.

Note that the use of a Bernoulli–Euler plate impedance is not justified in the case of a thick plate. When $\Omega > 1$, the results are not valid. Toplosky used it to avoid divergences in his formulation encountered with a Timoshenko–Mindlin plate impedance. The root of the problem was his use of a force dipole line moment, which cannot be supported by a Timoshenko–Mindlin plate (Rumerman, 1979).

INTEGRALS

The Green's functions are evaluated using contour integration. There are branch points at $\xi = \pm 1$ as well as poles in the complex ξ-plane, some on the real axis. The L-shaped Sommerfeld branch cuts are used. The results may be expressed as

$$\Gamma_J = 2\pi i \sum \text{residues}_J - I_{35,J} - I_{26,J} \, ,$$

where

$$I_{35,J}(d) = -\frac{k}{\pi} \int_0^1 (ik\xi)^J \frac{\rho c\sqrt{1-\xi^2}}{\left(\omega\rho_p h\right)^2 \left(\Omega^2\xi^4 - 1\right)^2 \left(1-\xi^2\right) + \left(\rho c\right)^2} e^{ikd\xi} d\xi \, ,$$

$$I_{26,J}(d) = i\frac{k}{\pi} \int_0^\infty (-k\xi)^J \frac{\rho c\sqrt{1+\xi^2}}{\left(\omega\rho_p h\right)^2 \left(\Omega^2\xi^4 - 1\right)^2 \left(1+\xi^2\right) + \left(\rho c\right)^2} e^{-kd\xi} d\xi \, .$$

The advantages of these expressions for numerical integration may be seen. In the case of an oscillatory integrand, the interval is simply [0,1]. In the case of an infinite interval, the integrand has exponential damping. The upper limit may therefore be set to a reasonable value (Toplosky uses 8).

NUMERICAL CONSIDERATIONS

Toplosky calculates the Green's functions using simple Romberg integration. This is sufficient at low frequencies. However, at high frequencies, the rational factor has a sharp peak near $\xi = \Omega^{-1/2}$ (width $\Delta\xi \propto \Omega^{-3/2}$), which may cause convergence problems, especially in combination with a rapidly oscillating exponential factor. Variable step size integration, such as in ordinary differential equation solvers, will provide superior performance. For $I_{26,J}$, when $d \neq 0$, the decaying exponential factor negates the significance of the peak.

HIGH-FREQUENCY APPROXIMATIONS

Based on the previous considerations, the integrals may be approximated when $\Omega \gg 1$. Again, note that the Toplosky formulation is not valid when $\Omega > 1$. However, despite this, we will find some useful results, as will be seen later.

We will begin with $I_{35,J}$ first. Change the integration variable to $z = \sqrt{\Omega \xi}$, so

$$I_{35,J}(d) = -\frac{k}{\pi}\frac{\rho c}{\sqrt{\Omega}}\left(\frac{ik}{\sqrt{\Omega}}\right)^J \int_0^{\sqrt{\Omega}} z^J \frac{\sqrt{1 - \frac{z^2}{\Omega}}}{\left(\omega\rho_p h\right)^2\left(z^4 - 1\right)^2\left(1 - \frac{z^2}{\Omega}\right) + \left(\rho c\right)^2}\exp\left(i\frac{kd}{\sqrt{\Omega}}z\right)dz ,$$

and the peak is now at $z = 1$. Note that $z^4 - 1 = \left(z^2 + 1\right)\left(z + 1\right)\left(z - 1\right) \approx 4\left(z - 1\right)$ in the vicinity of the peak, and that z^2 / Ω is negligible there as well. Therefore,

$$I_{35,J}(d) \approx -\frac{\rho c k}{\pi}\frac{\sqrt{\Omega - 1}}{\Omega}\left(\frac{ik}{\sqrt{\Omega}}\right)^J \int_0^{\sqrt{\Omega}} \frac{\exp\left(i\frac{kd}{\sqrt{\Omega}}z\right)dz}{\left[4\omega\rho_p h\sqrt{1 - \frac{1}{\Omega}}\right]^2\left(z - 1\right)^2 + \left(\rho c\right)^2} ,$$

$$\approx -\frac{k}{\pi}\frac{\exp\left(i\frac{kd}{\sqrt{\Omega}}\right)}{4\omega\rho_p h\sqrt{\Omega}}\left(\frac{ik}{\sqrt{\Omega}}\right)^J \int_{-y_0}^{y_0\sqrt{\Omega}} \frac{\exp\left(i\frac{kd}{\sqrt{\Omega}}\frac{y}{y_0}\right)dy}{y^2 + 1} ,$$

where $y_0 = 4kh\frac{\rho_p}{\rho}\sqrt{1 - 1/\Omega} = 8\sqrt{3}\frac{\rho_p c}{\rho c_p}\Omega\sqrt{1 - 1/\Omega} \gg 1$ for all practical problems. Thus, taking the integration limits as $\pm\infty$, we finally find

$$I_{35,J}(d) \approx -\frac{\exp\left(-\frac{\rho d}{4\rho_p h\sqrt{\Omega}}\right)}{4\rho_p ch\sqrt{\Omega}}\exp\left(i\frac{\omega_c d\sqrt{\Omega}}{c}\right)\left(i\frac{\omega_c\sqrt{\Omega}}{c}\right)^J .$$

In considering approximations to $I_{26,J}$, we must examine the case $d = 0$ separately.

$$I_{26,J}(0) = i\frac{k}{\pi}\frac{\rho c}{\sqrt{\Omega}}\left(-\frac{k}{\sqrt{\Omega}}\right)^J \int_0^{\infty} z^J \frac{\sqrt{1 + \frac{z^2}{\Omega}}}{\left(\omega\rho_p h\right)^2\left(z^4 - 1\right)^2\left(1 + \frac{z^2}{\Omega}\right) + \left(\rho c\right)^2} dz .$$

The development follows that for $I_{35,J}$,

$$I_{26,J}(0) \approx i\frac{k}{\pi}\frac{1}{4\omega\rho_p h\sqrt{\Omega}}\left(-\frac{k}{\sqrt{\Omega}}\right)^J \int_{-y_0}^{\infty} \frac{dy}{y^2 + 1} ,$$

where $y_0 = 4kh\frac{\rho_p}{\rho}\sqrt{1 + 1/\Omega} = 8\sqrt{3}\frac{\rho_p c}{\rho c_p}\Omega\sqrt{1 + 1/\Omega} \gg 1$. Thus,

$$I_{26,J}(0) \approx i\frac{1}{4\rho_p ch\sqrt{\Omega}}\left(-\frac{\omega_c\sqrt{\Omega}}{c}\right)^J .$$

When $d \neq 0$, the main contribution to $I_{26,J}$ comes from a region where $\xi \ll 1$. Therefore,

$$I_{26,J}(d) \approx i\frac{k}{\pi}\frac{\rho c}{\left(\omega\rho_p h\right)^2 + \left(\rho c\right)^2}(-k)^J \int_0^\infty \xi^J e^{-kd\xi} d\xi ,$$

$$\approx i\frac{1}{\pi d}\frac{\rho c}{\left(\omega_c \rho_p h\Omega\right)^2 + \left(\rho c\right)^2}\left(-\frac{1}{d}\right)^J (J!) .$$

EXAMPLE

Let us consider a 5-cm-thick steel plate (coincidence frequency of 4.7 kHz). The "exact" and approximate results are compared in figures 1 through 9. We find that the approximation of $I_{35,J}$ is accurate (within 10%) for $\Omega > 2$. The approximation of $I_{26,J}$ is accurate for $\Omega > 0.2$. It is this result that makes the analysis worthwhile. Similar results are found for a 2.5-cm-thick steel plate (coincidence frequency of 9.4 kHz).

SUMMARY

Approximations to integrals in the Toplosky rib formulation have been found. Though derived for high frequencies, many are useful over a much broader range. These approximations save a great deal of computation time.

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Rumerman, M. L. 1979. "Nonspecular Acoustic Backscattering from Finite Plates," *Journal of the Acoustical Society of America*, vol. 65, pp. 1121–1126.

Stepanishen, P. R. 1978. "The Acoustic Transmission and Scattering Characteristics of a Plate with Line Impedance Discontinuities," *Journal of Sound and Vibrations*, vol. 58, pp. 257–272.

Toplosky, N. and R. D. Vogelsong. 1993. "The Influence of Rib Interaction on the Acoustic Scattering from an Elastic Plate," NUWC–NPT Technical Report 10,349, (Sep), Naval Undersea Warfare Center, Newport, RI.
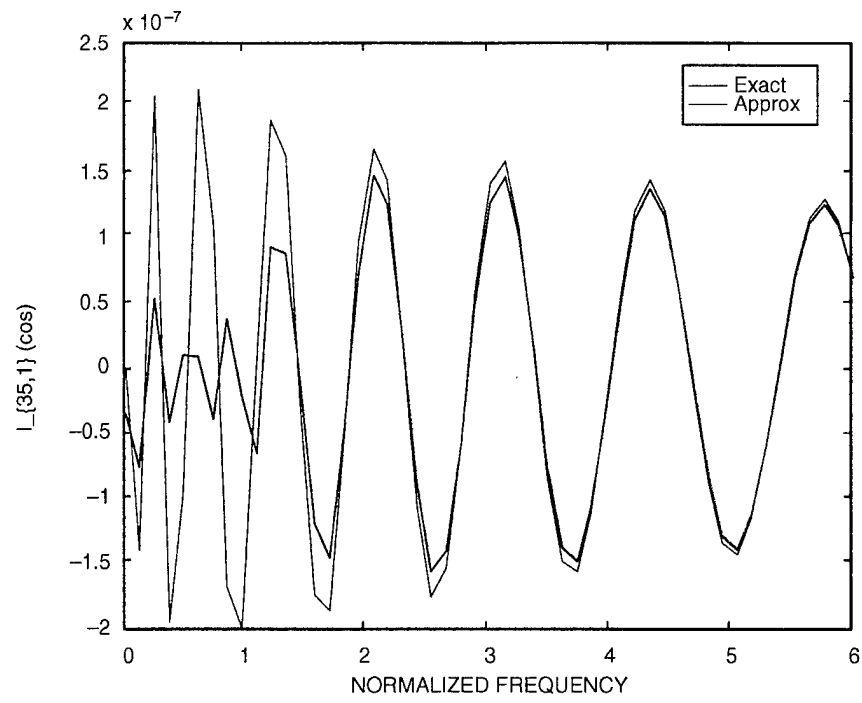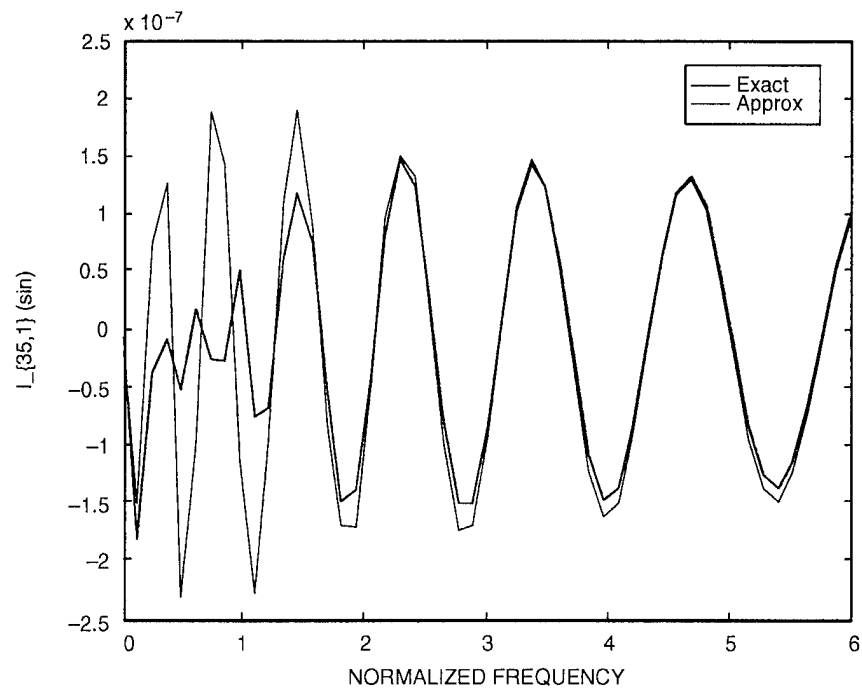
Figure 1. Comparison for $I_{35,1}$ (cosine part).

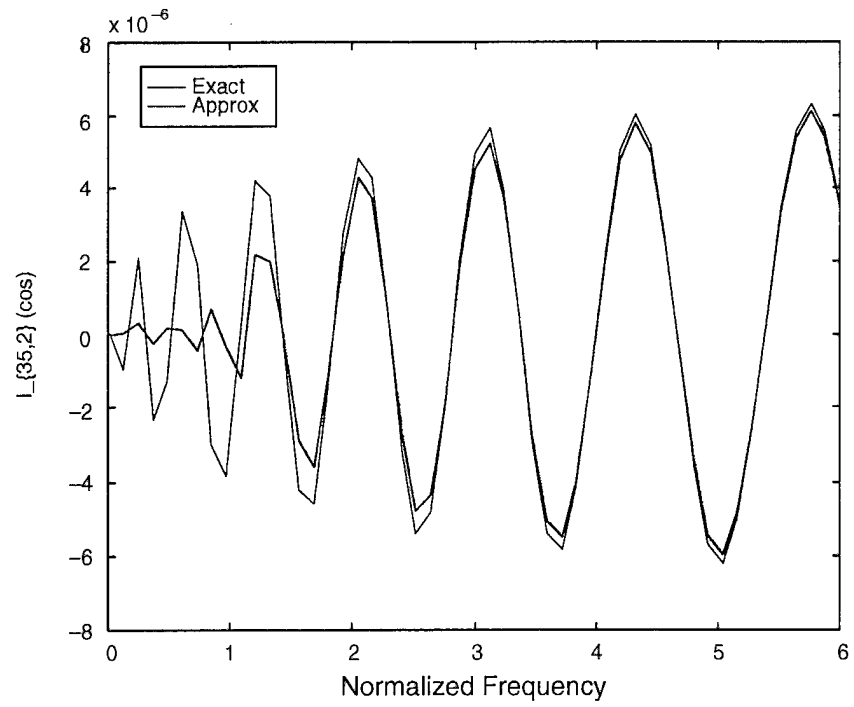Figure 2. Comparison for $I_{35,1}$ (sine part).

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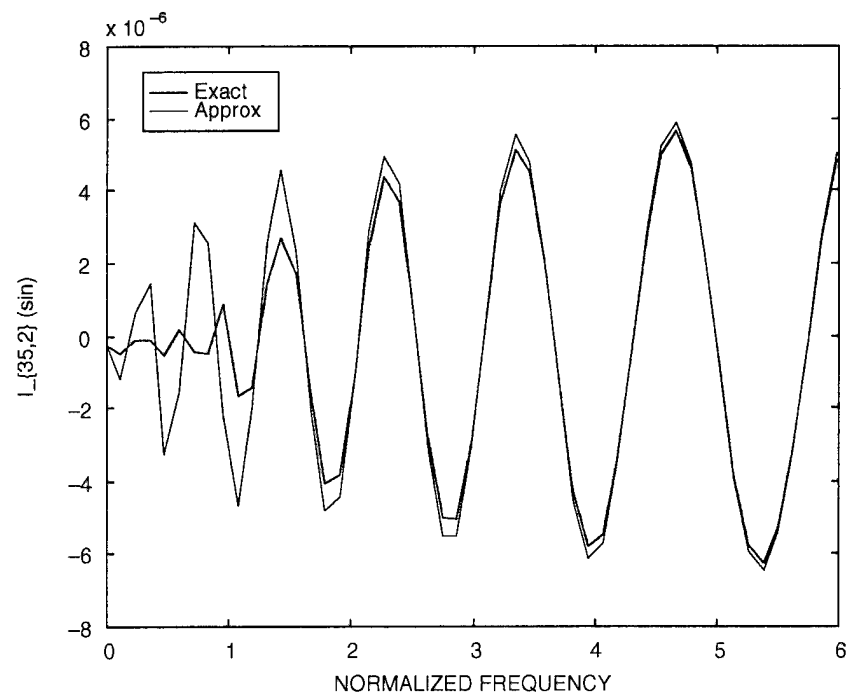Figure 3. Comparison for $I_{35,2}$ (cosine part).

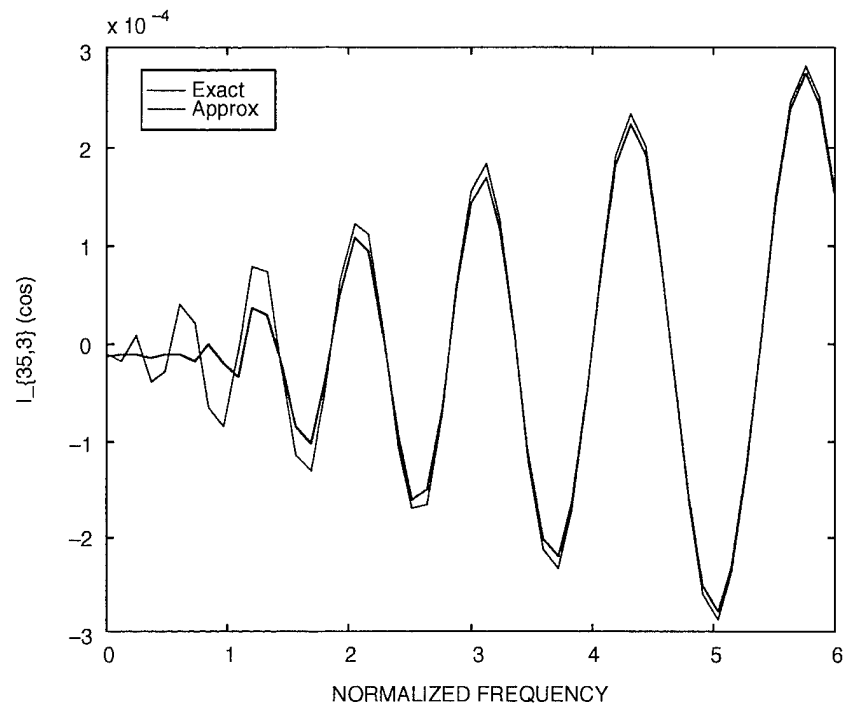Figure 4. Comparison for $I_{35,2}$ (sine part).

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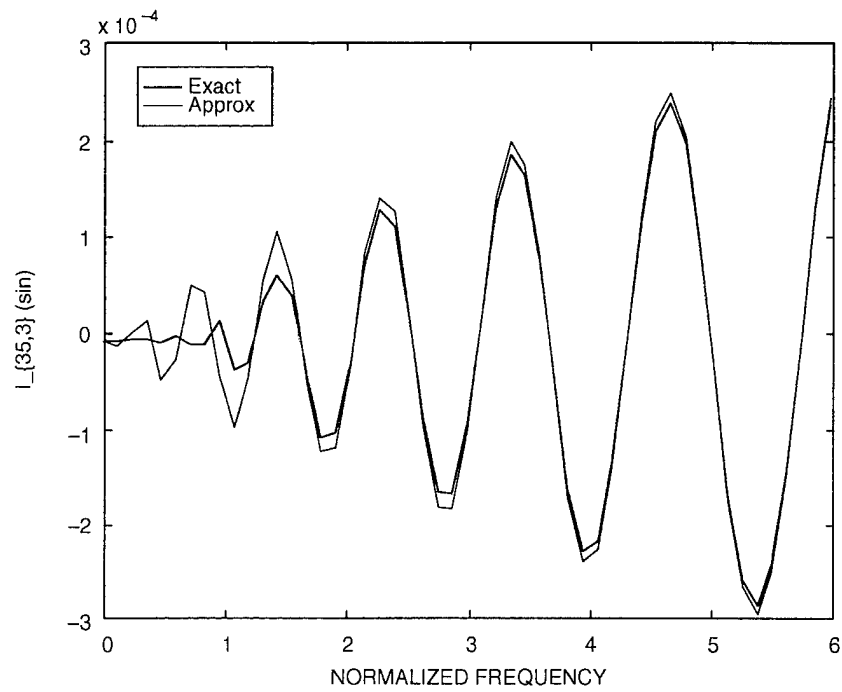Figure 5. Comparison for $I_{35,3}$ (cosine part).
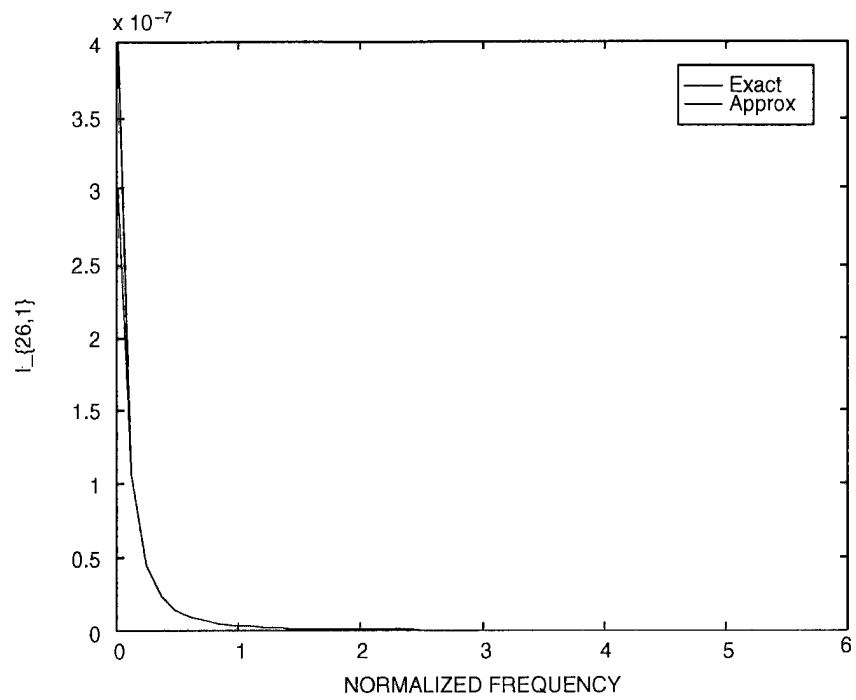

Figure 6. Comparison for $I_{35,3}$ (sine part).
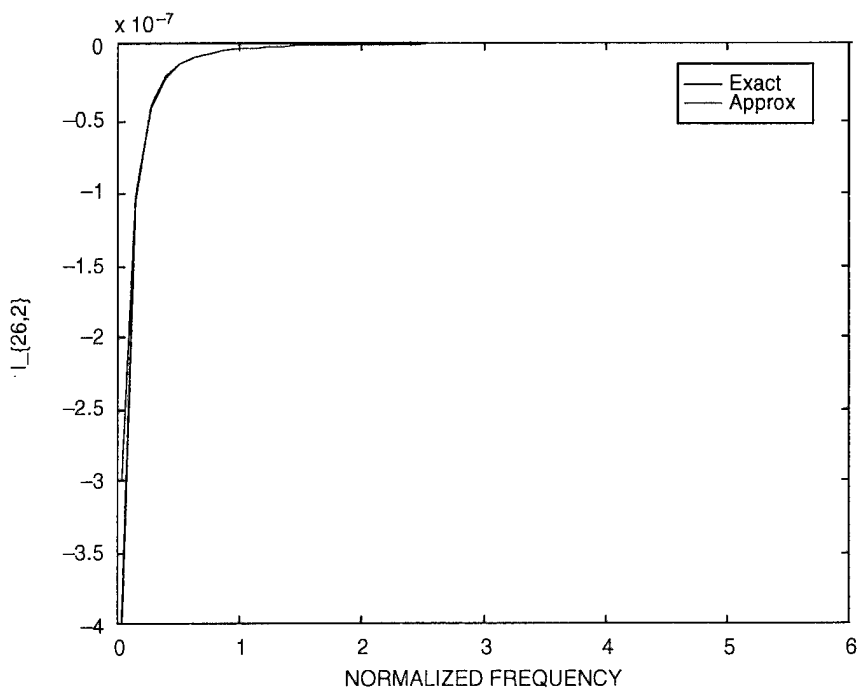
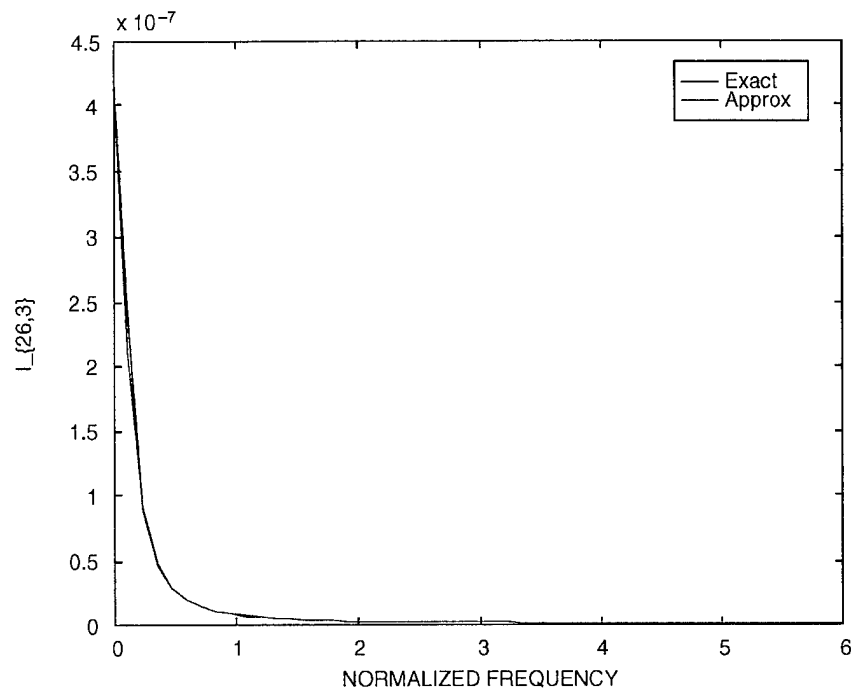Figure 7. Comparison for $I_{26,1}$.

Figure 8. Comparison for $I_{26,2}$.

Figure 9. Comparison for $I_{26,3}$.

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